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A refresher on a quality tool, concept or method

Are You Confident?

Calculating confidence intervals by Peter Sherman

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ow do you assess how much you really know when so much is unknown? The answer is confidence intervals. This is illustrated by the trivia quiz in Online Table 1, which can be found on this column's webpage at qualityprogress.com. You can do well on the quiz without

knowing the actual answers to any of the questions. For each question, provide a low and high guess such that you are 90% sure the correct answer falls between the two. If you succeed, you should have nine correct answers and only one wrong answer. That is, your answers should be correct 90% of the time.

The answers are provided in Online Table 2. How many did you get correct? Were you surprised? You're probably thinking, "Why does it matter whether I know any of this random trivia? I shouldn't care if my confidence intervals are wrong because the questions are outside of my expertise."

But you should care. These trivia questions measure how well you know what you don't know. If you had a lot of domain knowledge about the subject of one of the questions, your 90% confidence level would be narrow. If you knew less, it would be wider. Either way, your 90% confidence interval, by definition, should have captured the true answers 90% of the time.

Confidence intervals allow you to estimate population parameters (such as the mean, standard deviation and proportions) with a known degree of certainty or confidence. When unsure about a decision, confidence intervals are better than providing a single-point estimate. The confidence interval is bounded by a lower and upper limit, which are determined by the risk associated with making a wrong conclusion about the parameter of interest. This is known as alpha risk and is stated in terms of probability.

For example, a 95% confidence interval equates to a 0.05 or 5% alpha risk (1 - confidence interval). For a 95% confidence interval, the area in each tail is 0.025 (0.05 / 2) (see Online Figure 1). With a normal distribution, the empirical rule states that 68% of the values lie within one standard deviation of the mean, 95% within two standard deviations and 99.73% within three standard deviations (see Online Figure 2).

There are five steps to calculating confidence ranges:

1. Collect historical data (n).
2. Calculate the mean (X). The Excel formula is "=average(A1:A10)."
3. Calculate the standard deviation (s). The Excel formula is "=stdev(A1:A10)."

4. Select the appropriate Z value (number of standard deviations) based on your confidence level (see Online Table 3).
5. Calculate the confidence ranges using the formula:

$$\begin{array}{ccc} \text{Lower range} & & \text{Lower range} \\ \bar{X} - Z \left(\frac{s}{\sqrt{n}} \right) & \text{---} & \bar{X} + Z \left(\frac{s}{\sqrt{n}} \right) \\ & & \bar{X} \end{array}$$

Example

During Thanksgiving break one year, I asked my son how he thought he would do on his calculus II final exam in college. He said, "I think I'll get an A, but I'm not very sure."

I rephrased the question: "What's your confidence range estimate on the final exam, assuming a 95% confidence level?"

During the semester, he took four calculus exams (n = 4) with the following scores: 84, 85, 92 and 87. We calculated the mean and standard deviation at 87 and 3.56, respectively. Because the sample size was less than 30, we used the student's t table, a more conservative variation of the Z table. The t value for a 95% confidence level (2-tailed) based on three degrees of freedom (n - 1) is 3.1824 (see Online Figure 3).

"Degrees of freedom" is a statistical term that refers to the number of independent observations in a sample minus the number of population parameters, which, in this case, was one.¹ Plugging the values into our formula produced a lower limit of 82 and an upper limit of 93.

My son then said: "Dad, based on my previous scores, I estimate my final score on the calculus final will fall somewhere between 82 and 93. I am 95% confident."

I responded: "Michael, you're a fast learner. But let's try for the upper range, please." **QP**

REFERENCE

1. "Degrees of Freedom (statistics)," Wikipedia, <https://tinyurl.com/y6uybwfw>.



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